arXiv:hep-th/9606176v1 27 Jun 1996

Multiple Realisations of $\mathcal{N}{=}1$ Vacua in Six Dimensions

Eric G. Gimon[a] **and Clifford V. Johnson**[b]

[a]*Department of Physics /* [b]*Institute for Theoretical Physics,*

University of California,

Santa Barbara, CA 93106, USA

Abstract

A while ago, examples of $\mathcal{N}{=}1$ vacua in $D{=}6$ were constructed as orientifolds of Type IIB string theory compactified on the $K3$ surface. Among the interesting features of those models was the presence of D5–branes behaving like small instantons, and the appearance of extra tensor multiplets. These are both non–perturbative phenomena from the point of view of Heterotic string theory. Although the orientifold models are a natural setting in which to study these non–perturbative Heterotic string phenomena, it is interesting and instructive to explore how such vacua are realised in Heterotic string theory, M–theory and F–theory, and consider the relations between them. In particular, we consider models of M–theory compactified on $K3{\times}S^1/\mathbf{Z}_2$ with fivebranes present on the interval. There is a family of such models which yields the same spectra as a subfamily of the orientifold models. By further compactifying on T^2 to four dimensions we relate them to Heterotic string spectra. We then use Heterotic/Type IIA duality to deduce the existence of Calabi–Yau 3–folds which should yield the original six dimensional orientifold spectra if we use them to compactify F–theory. Finally, we show in detail how to take a limit of such an F–theory compactification which returns us to the Type IIB orientifold models.

June, 1996

[a]`egimon@physics.ucsb.edu`, [b]`cvj@itp.ucsb.edu`

1. Introduction

In six dimensions, vacua with $\mathcal{N}=1$ supersymmetry have a rich and interesting structure. Due to potential chiral anomalies, such vacua are subject to constraints which enable us learn a great deal about the nature of the various sibling string theories (and their parent theories) which can give rise to them.

Due to various perturbative and non–perturbative symmetries, a given spectrum may be obtained in many different ways. For example, by studying $SO(32)$ heterotic string vacua in $D{=}6$, using constraints from the chiral anomalies, the presence of non–perturbative effects attributable to small instantons may be deduced[1]. In the dual type I theory, these non–perturbative effects can be studied perturbatively as D5–branes[1,2]. This is a consequence of ten dimensional strong/weak coupling duality between the two $SO(32)$ theories[3].

Another example is heterotic/heterotic duality[4]. The conjecture about the existence of this duality was motivated[5] in part by the structure of the factorised 8–form polynomial associated to the anomaly of a $D{=}6$ heterotic $K3$ compactification[1]. The realisation of the possibility of non–perturbative gauge groups due to small instantons allowed the conjecture to be confirmed[4] in terms of a $K3$ compactification of the $E_8{\times}E_8$ heterotic string with a choice of vacuum gauge bundle with 12 instantons assigned to each E_8. The conjectured duality map acts non–trivially on the gauge group and hypermultiplets to exchange the perturbative and non–perturbative contributions to this $D{=}6$ string vacuum.

In an apparently (at the time) different setting, an orientifold construction of type I string theory compactified on $K3$ to six dimensions was presented[2]. The $K3$ manifold was in its $T^4/\mathbf{Z}_2$ orbifold limit. Upon closer examination, a relation between (a special case of) that model and the heterotic/heterotic construction may be deduced[6]: This model, constructed using perturbative string techniques, enjoys the presence of two isomorphic sectors to its gauge group (and charged hypermultiplets), attributable to the necessary presence of both D9– and D5–branes. The isomorphism between the two sectors is simply realised as perturbative T–duality in the X^6, X^7, X^8 and X^9 directions of the $K3$ torus (denoted T_{6789}–duality henceforth).

As this model is a compactification of $SO(32)$ type I string theory, it ought to be strong/weak coupling dual to a compactified heterotic model, by virtue of their relationship[3] in $D{=}10$. This turns out to be true, and the details are very instructive. It is the $E_8{\times}E_8$ $(12,12)$ compactification which turns out to be relevant, with the subtlety residing in the fact that what were naively small $SO(32)$ instantons from the type I

---

[1] Amusingly, it seems that it was almost rejected for the same reason, as the signs of some of the $\text{Tr}F^2$ terms in one of the factors (for the standard embedding) signaled that there would be unpleasant behaviour somewhere in coupling space. This is now interpreted as the sign of a phase transition[4].

perspective, or small E_8 instantons from the heterotic perspective, are actually properly thought of as $spin(32)/\mathbf{Z}_2$ instantons for that particular embedding[6]. Under this particular type I/heterotic map, the perturbative T_{6789}–duality of the type I string induces the conjectured heterotic/heterotic strong/weak coupling duality map in the heterotic picture. Along the way, we learn again that the distinction between the two types of heterotic string is weakened when we leave ten dimensions, this time compactifying on $K3$.

This heterotic model also has a realisation in F–theory. This twelve dimensional setting, which may be regarded as a powerful means of generating new consistent backgrounds for the type IIB string, gives rise to $\mathcal{N}{=}1$ vacua in $D{=}6$ after compactification on an elliptically fibred Calabi-Yau three–fold[7]. In fact, it has been shown that the family of Calabi–Yau manifolds which may be described as a fibration of a torus over the 'ruled surfaces' F_n, are the appropriate elliptic 3–folds on which to compactify F–theory in order to realize duals to the heterotic vacua obtained by compactifying on $K3$ with instanton embedding $(12{-}n, 12{+}n)$. In the case $n{=}0$, the surface F_0 is simply the product of two–spheres $\mathbb{P}_1{\times}\mathbb{P}_1$. The elliptic 3–fold is the ubiquitous degree 24 hypersurface in weighted projective space $\mathrm{W}\mathbb{P}(1,1,2,8,12)$, denoted $X_{24}(1,1,2,8,12)$. The heterotic/heterotic duality map of the (12,12) model[2] becomes the exchange of the two $\mathbb{P}_1$'s, one carrying the data to be labeled as perturbative in the heterotic model and the other $\mathbb{P}_1$ carrying the non–perturbative data. As for the orientifold setting, this is another arena in which the perturbative and non–perturbative structures (from the point of view of the heterotic string) are treated on the same footing. We shall study this particular model some more in section 6, establishing a direct relation between the orientifold and F–theory constructions, following the ideas in ref.[10].

The purpose of this paper is to try to understand more of such specific examples of $\mathcal{N}{=}1$ vacua in six dimensions in many different settings. To this end, we shall follow a circular chain of dualities studying special cases of some of the orientifold models presented in refs.[11,12] which are closely related to the orientifold model discussed above.

We start by considering M–Theory. The strong coupling limit of ten dimensional $E_8{\times}E_8$ heterotic string theory is M–theory on the orbifold $S^1/\mathbf{Z}_2$. It is (arguably) unnecessary to go to this eleven dimensional theory to explain even the non–perturbative aspects of the $(12,12) = (10,14)$ $E_8{\times}E_8$ heterotic models, as they only require an appeal to small $spin(32)/\mathbf{Z}_2$ instantons and not small E_8 instantons. This will not remain true for the models we consider here. Specifically it is in M–theory that the non–perturbative appearance of extra tensor multiplets in the six dimensional spectra have a most natural description[13], and this shall be the starting point for this paper's study (section 3) when we come to try to learn more about the orientifold models we consider (recalled in section 2).

---

[2] The (10,14) model turns out to be the same model, as conjectured in ref.[8] and demonstrated in the F-theory context in ref.[9]. This is simply because the relevant elliptic fibration over F_2 is isomorphic to the one over F_0.

From the M–theory and heterotic string discussions in six dimensions, we compactify further to four (in section 4), where in that $\mathcal{N}{=}2$ context, the two theories are indistinguishable from each other, as the extra tensors give rise to vectors in four dimensions. Next, we use heterotic/type IIA strong/weak coupling duality to take us to type II string theory, by conjecturing the existence of two new (to us, at least) $K3$ fibred Calabi–Yau 3–folds suggested by the spectra.

This leads us to consider returning to type IIB theory (in section 5), and we do so by going to F–theory, asking that we can compactify it on the 3–folds. If they have an elliptic fibration as well, we can construct six dimensional $\mathcal{N}{=}1$ vacua yielding the spectra we first started with.

To close the circle, we should be able to find a limit of F–theory which directly gives a perturbative type IIB background, and indeed there is one. Extending the ideas in ref.[10] in section 6 we show how to recover from F–theory orientifold models which are simple T–duals of the ones we started with.

Our circular route is summarised below:
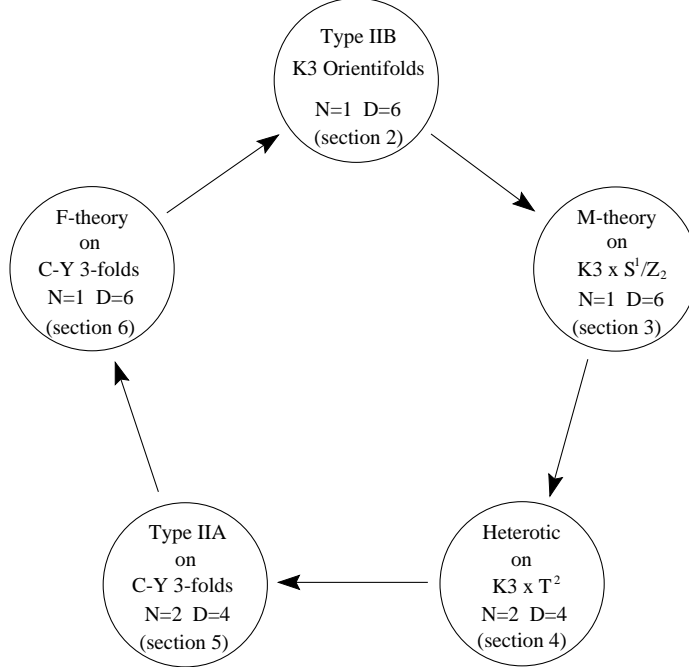
There are three specific models which we take around the path we just described. They are

all quite different models in the orientifold context in which they were first presented[3][11][2]. Once we get to M–theory we see that they are probably related by rather simple phase transitions of the nature discussed in refs.[14,15,16]. From our comments about F–theory, they are likely to be closely related in that context, although the issues are clouded somewhat by our ignorance about the Calabi–Yau 3–folds which we conjecture to exist. As a result, we complete the discussion of the connection of F–theory with the orientifold models in detail only for the one related to the heterotic/heterotic model, leaving the details of the other models for the time when we have more data on the Calabi–Yau 3–folds.

---

[3] The first example (originating in ref.[2]) has been taken through most of this discussion previously, as mentioned above. However, the details of the direct F–theory connection are worked out here for the first time.

2. The Orientifold Models

Let us briefly recall the models presented in ref.[11]. They are constructed as two types of orientifold of type IIB strings which give rise to six dimensional $\mathcal{N}=1$ supersymmetric models, with the compact internal spacetime being the torus T^4 with orbifold identifications. In the terminology of ref.[11], type 'A' models contain the explicit appearance of the world–sheet parity operator Ω, and results in tadpoles which must be cancelled with D9–branes. Type 'B' models do not contain Ω. The worldsheet operation Ω is consistently (at the level of perturbation theory[6]) combined with discrete spacetime rotation symmetries, $\mathbf{Z}_N$ which on their own, in a closed string setting, would have resulted in an orbifold. The combination of the two results in an 'orientifold', and requires the addition of open string sectors, personified in the form of 'D–branes' to cancel the resulting tadpoles. Depending upon the order of the rotation symmetry used for the spacetime symmetry, there may be the requirement to introduce D5–branes for this tadpole cancellation, with their world volumes aligned with the non–compact directions $\{X^0, \ldots, X^5\}$.

For the particular choice of realisation of the symmetries $\mathbf{Z}_N$ and Ω used in the work of ref.[11], the models which arose (denoted[4] $\mathbf{Z}_3^A, \mathbf{Z}_4^A, \mathbf{Z}_4^B, \mathbf{Z}_6^A, \mathbf{Z}_6^B$ and $\mathbf{Z}_2^A$) had the following closed string $\mathcal{N}=1$ $D=6$ spectra[5] (in addition to the usual supergravity multiplet and tensor multiplet):

Model	Neutral Hypermultiplets	Extra Tensor Multiplets
$\mathbf{Z}_2^A$	20	0
$\mathbf{Z}_3^A$	11	9
$\mathbf{Z}_4^A$	16	4
$\mathbf{Z}_6^A$	14	6
$\mathbf{Z}_4^B$	12	8
$\mathbf{Z}_6^B$	11	9

With the exception of the $\mathbf{Z}_2^A$ model, which is the model of ref.[2], all of the models have extra self–dual tensor multiplets (the perturbative heterotic stirng has only one), and a reduced (from the standard 20) number of hypermultiplets corresponding to the gravitational moduli of the $K3$ surface upon which we have compactified.

---

[4] The $\mathbf{Z}_2^A$ model was constructed in ref.[2], and a special case of it is the dual of the (12,12) heterotic model, as discussed above and later.

[5] Throughout the paper, due to the presence of supersymmetry it is enough to display only the bosonic part of the spectra we consider.

In view of this, we ought not to think of these models with extra tensors as compactifications of the type I string on some $K3$ manifold, as we will run into trouble for many related reasons, some of which we list below:

1. The origin of the extra tensors would be problematic in that setting. This is simply because geometrically they would need to have a ten dimensional origin in a self–dual 4–form A_4 of the R-R sector of the theory, as this is the only object which could be contracted with the two–cycles of $K3$ to give self–dual 2–forms in six dimensions. However, in obtaining type I from type IIB via the simple orientifold with Ω in ten dimensions, the 4–form A_4, which is odd under Ω, gets projected out of the theory and leaves us with no candidate to give rise to the extra tensors after compactifying.

2. If this was the limit of a smooth type I compactification on $K3$, there should be a limit where we could enlarge the $K3$ manifold to get an effective ten dimensional theory, use strong/weak coupling duality to get an $SO(32)$ heterotic theory, and then shrink $K3$ again, thus deducing six dimensional dual $SO(32)$ heterotic theories which seem to have extra tensor multiplets arising apparently at all values of the coupling.

This latter reason by itself is not so unsettling a problem, as we have learned in recent times to be open to the idea of the appearance of new structures which cannot be seen perturbatively, but persist at all values of the coupling. However, it is all too easy to be satisfied with such an explanation without exploring it further. In particular, the deduction that small instantons give rise to new non–perturbative phenomena[1] in the heterotic string arose by considering[17] a singular limit of a perturbatively well–defined heterotic object, the solitonic fivebrane instanton[18]. In some loose sense then, there was some herald of such peculiar non–perturbative behaviour in perturbative heterotic string theory. In this context, the above (bogus) deduction (**2**) would lead us to deduce new non–perturbative structures in the $SO(32)$ heterotic string —the extra tensors— for the wrong reasons, and we would not have had a suggestion (analogous to the big fivebrane instantons) that such phenomena might appear.

So the resolution of the points are as follows:

1′. The operations of orientifolding and compactifying do not commute, in general. The extra tensors arise simply because A_4 has not been projected with Ω, but with $\widetilde{\Omega}$, which is the combination of Ω with a spacetime symmetry under which (components of) A_4 transforms[12,19]. It therefore can survive in the resulting model, contracting with $K3$ two–cycles and giving rise to self–dual 2–forms in the six dimensional model. The choice which was implemented for the operation of Ω in the closed string sector implied in ref.[11] was implicitly a realisation of type[6] $\widetilde{\Omega}$.

2′. As a result of the choices made, the reduced set of moduli for the $K3$ surface result

---

[6] Julie Blum has also computed explicit realisations of the action of Ω and $\widetilde{\Omega}$ in the closed string sectors of explicit orientifold models.

in it being impossible[19] to blow up the $\mathbf{Z}_N$ fixed points (for $N>2$) to completely smooth manifolds for the generic models[7]. This prevents us from being able to glibly deduce perturbative heterotic models as duals along the lines indicated above. The naive deduction via the scenario (**2**) above would have led to only an $SO(32)$ (or at best, $spin(32)/\mathbf{Z}_2$) framework. The phenomenon of extra tensors fits more naturally into an $E_8 \times E_8$ setting, which can be either found in the $E_8 \times E_8$ heterotic string, or something closely related to it — M–theory on an $S^1/\mathbf{Z}_2$ orbifold[20].

As shown in refs.[2,11] the open string spectrum produced massless vector and hypermultiplets resulting in the gauge content listed in the table below[8]:

Model	Gauge Group		Charged Hypermultiplets	
$\mathbf{Z}_2^A$	99 :	$U(16)$	99 :	$2 \times \mathbf{120}$
	55 :	$U(16)$	55 :	$2 \times \mathbf{120}$
			59 :	$(\mathbf{16}, \mathbf{16})$
$\mathbf{Z}_3^A$	99 :	$U(8) \times SO(16)$	99 :	$(\mathbf{28}, \mathbf{1}); \ (\mathbf{8}, \mathbf{16})$
$\mathbf{Z}_4^A$	99 :	$U(8) \times U(8)$	99 :	$(\mathbf{28}, \mathbf{1}); \ (\mathbf{1}, \mathbf{28}); \ (\mathbf{8}, \mathbf{8})$
	55 :	$U(8) \times U(8)$	55 :	$(\mathbf{28}, \mathbf{1}); \ (\mathbf{1}, \mathbf{28}); \ (\mathbf{8}, \mathbf{8})$
			59 :	$(\mathbf{8}, \mathbf{1}; \mathbf{8}, \mathbf{1}); \ (\mathbf{1}, \mathbf{8}; \mathbf{1}, \mathbf{8})$
$\mathbf{Z}_6^A$			99 :	$(\mathbf{6}, \mathbf{1}, \mathbf{1}); \ (\mathbf{1}, \mathbf{6}, \mathbf{1})$
				$(\mathbf{4}, \mathbf{1}, \mathbf{8}); \ (\mathbf{1}, \mathbf{4}, \mathbf{8})$
	99 :	$U(4) \times U(4) \times U(8)$	55 :	$(\mathbf{6}, \mathbf{1}, \mathbf{1}); \ (\mathbf{1}, \mathbf{6}, \mathbf{1})$
	55 :	$U(4) \times U(4) \times U(8)$		$(\mathbf{4}, \mathbf{1}, \mathbf{8}); \ (\mathbf{1}, \mathbf{4}, \mathbf{8})$
			59 :	$(\mathbf{4}, \mathbf{1}, \mathbf{1}; \mathbf{4}, \mathbf{1}, \mathbf{1})$
				$(\mathbf{1}, \mathbf{4}, \mathbf{1}; \mathbf{1}, \mathbf{4}, \mathbf{1})$
				$(\mathbf{1}, \mathbf{1}, \mathbf{8}; \mathbf{1}, \mathbf{1}, \mathbf{8})$
$\mathbf{Z}_4^B$		—		—
$\mathbf{Z}_6^B$	55 :	$U(8) \times SO(16)$	55 :	$(\mathbf{28}, \mathbf{1}); \ (\mathbf{8}, \mathbf{16})$

---

[7] This is true within the orientifold framework. In M–theory we will see that the $K3$ we compactify on has the full set of moduli available.

[8] We list all of the charges of the hypermultiplets for completeness. The appearances of notation like '99', '55' and '59' mean massless fields arising from strings stretched between coincident D–branes of type 9 or 5.

It is interesting to see how the potential chiral anomalies of these models are cancelled. In particular, the irreducible $\mathrm{Tr}R^4$ and $\mathrm{Tr}F^4$ terms vanish, the former by virtue of each model solving the anomaly equation

$$n_H - n_V = 244 - 29n_T, \qquad (2.1)$$

where n_H, n_V and n_T are the number of hyper–, vector– and extra tensor–multiplets in a model (perturbative heterotic string theory has n_T=0). The remaining anomalies are cancelled by a generalisation of the Green–Schwarz mechanism[21,22,6], where all of the tensors in the model come into play, having one–loop couplings to the gauge and gravitational sectors, together with their field strengths being modified from the naive form to one in which gravitational and gauge Chern–Simons forms appear in the standard way.

Before moving on, let us recall a few things easily noticed about the models[11]. Notice that the $\mathbf{Z}_3^A$ model is isomorphic to the $\mathbf{Z}_6^B$ model. The map between them is the aforementioned T_{6789}–duality, which exchanges D9– with D5–branes. All of the other models presented in the table are self–dual under this operation. Of these, the $\mathbf{Z}_4^B$ model has no D–branes and thus no gauge sector. It is a purely closed string model.

The remaining models, $\mathbf{Z}_2^A, \mathbf{Z}_4^A$ and $\mathbf{Z}_6^A$, are all similar in some sense (which will be the main focus of this paper):

a. They all contain 32 of both D9– and D5–branes, which get exchanged under T_{6789}. The associated gauge groups and hypermultiplets get exchanged under this operation.

b. They all have gauge groups of the same rank, which are successively smaller subgroups of the original $U(16){\times}U(16)$ as N gets larger.

c. It was learned that away from fixed points, the D5–branes were constrained by the consistency conditions to move as a single unit made of $2N$ D5–branes, forming a 'dynamical fivebrane' which it is tempting to identify with the 'small instanton' unit in a dual heterotic model. Indeed, they give rise to the same enhanced gauge groups in bulk as the basic small instanton example[1], and various patterns of enhancements as they settle on fixed points[2,11].

d. The numbers of the dynamical fivebranes available in each model ($\mathbf{Z}_2^A, \mathbf{Z}_4^A$ and $\mathbf{Z}_6^A$), are 8, 4 and 2 respectively. Adding these to the numbers of extra tensors in each model, which are 0, 4 and 6 respectively, always results in the number 8.

These features should mean something in the final analysis, and we can make some guesses as to what they might suggest:

a′. In the case of the $\mathbf{Z}_2^A$ model, a relation to a heterotic model has been demonstrated in ref.[6]. For a particular arrangement of D5–branes which we will discuss later (and inclusion of Wilson lines) it is a realisation of the $E_8{\times}E_8$ heterotic string compactified on $K3$ with instanton embedding (12,12). The perturbative T_{6789}–duality, which acts

non–trivially on the vector– and hyper–multiplets coming from the D9– and D5–brane sectors, gets mapped to the heterotic/heterotic duality map, which acts non–trivially on the perturbative vector– and hyper–multiplets to exchange them with those appearing non–perturbatively due to small $spin(32)/\mathbf{Z}_2$ instantons.

In the blow–down limit of the $K3$ surface in the orientifold, the instanton number 24 is distributed amongst the 8 small instantons and the 16 fixed points of the orbifold which are the blowdown of Eguchi–Hanson spaces[23] $\mathcal{E}_2$.

So in the other models, when we find dual realisations of them, we can hope to find an understanding of what the operation T_{6789} maps to.

$\mathbf{b}'$. In all of the models we will choose the special case where the gauge group is generally completely broken. It will be these cases which have a relation to the M–theory models which we construct.

$\mathbf{c}'$ & $\mathbf{d}'$. There is some description of a 'parent model' in which a single type of object, of which there are 8 in total, are present for reasons of charge cancellation, or some other (perhaps topological) reason. We can move between different parts of the moduli space of this parent model, and in a dual orientifold setting, we realize one of models $\mathbf{Z}_2^A, \mathbf{Z}_4^A$ or $\mathbf{Z}_6^A$, depending upon the details.

Let us try to discover the nature of this parent model.

3. M–Theory

M–theory is the first setting in which we shall try to fit the rest of our models. As discussed before, it is not necessary to appeal to M–theory to understand the $\mathbf{Z}_2^A$ model, but when we try to understand the extra tensors in the other models it is very natural.

The strong coupling limit of $E_8 \times E_8$ heterotic string theory in ten dimensions has been shown[20] to be a simple 'orbifold' of the eleven dimensional M–theory, where the eleventh dimension is placed on an orbifold $S^1/\mathbf{Z}_2$. Not much is known about orbifolds of this still unknown theory, but whatever happens should of course not contradict results we know to be true in string and field theory. In this spirit, the authors of ref.[20] showed that the ten dimensional spacetime living at each end of the line segment resulting from the orbifold should give rise to an E_8 gauge group, which give rise to the $E_8 \times E_8$ of the heterotic string in the weak coupling limit (when the size of the $S^1/\mathbf{Z}_2$ goes to zero).

A consistent heterotic string compactification on $K3$ requires[24] a choice of a gauge bundle of instanton number 24. This choice can be split between the factors of the gauge group in a way labeled by the integer n, placing instanton number $12-n$ in the first E_8 and $12+n$ in the other. These $(12-n, 12+n)$ embedding models have been discussed extensively in the literature recently. The spectrum resulting from this embedding is determined by an index theorem[24] which yields the number of hypermultiplets in the **56** of the resulting

E_7 gauge group (for a choice of $SU(2)$ instanton bundles). In particular, for $n=0, 2$ we have enough **56**'s, to allow us to break the gauge group completely, by sequential use of the Higgs mechanism. (As mentioned before, the cases $n=0$ and 2 turn out to be related to one another.) This results in a number of uncharged hypermultiplets, which must equal 244, by the anomaly equation (2.1).

The special case of the $\mathbf{Z}_2^A$ model[2] related to this model is as follows: It is possible to arrange the D5–branes (and by T_{6789}-duality, introduce Wilson lines) in such a way as to break the gauge symmetry carried by the D5–branes (D9–branes) completely. This is done by placing two D5–branes at each of the 16 $\mathbf{Z}_2$ fixed points. Naively, this yields a gauge group $U(1)^{16}$, according to the analysis of ref.[2], but the work of ref.[6] shows that such $U(1)$ groups are broken. This again results in no vectors and a number of uncharged hypermultiplets, which must equal 244, by the anomaly equation (2.1).

This unique configuration has at least one other interesting property of note. The $\mathbf{Z}_2$ fixed points carry -2 units of charge in D5–brane units. This is one way to determine that there are 32 D5–branes ($= 8$ dynamical fivebranes) available in the problem, for consistency. By the arrangement above, not only is charge cancellation satisfied in the compact space, but further to that it is satisfied locally. This statement applies (by supersymmetry) to the cancellation for the NS-NS sector, implying in particular that the dilaton has been cancelled locally. This in turn assures us that a perturbative heterotic dual can be found[25]. Other configurations where the charges were not cancelled locally would have regions where the dilaton could approach values corresponding to regions of strong coupling in any dual picture, thus spoiling a strong/weak coupling duality construction.

Notice also that as the minimal fivebrane object allowed to move in the bulk (*i.e.*, off the fixed points) is a collection of 4 D5–branes, this configuration has no flat directions corresponding to un–Higgs–ing back to the generic gauge groups found in ref.[2], as there are half–fivebranes on each distinct point. At this level, there is nothing to rule out the possibility of another Higgs–ing route to another branch with different gauge groups. One such branch available is the E_7 one of the $K3$ compactified $(12, 12)$ heterotic model to which this has been shown to be equivalent[6].

Something new arises when we realize that there are other ways of constructing consistent M–theory compactifications[26,13]. The consistency condition can be thought of as a charge cancellation condition for the 6–form potential in M-theory, obtained by dualising the 3–form. (This is related of course to the R-R 6–form charge cancellation condition of the orientifold model). More properly, there is the usual ten dimensional correction to the field strengths from both the geometry and gauge sectors in order to cancel the anomaly. The eleven dimensional theory is anomaly free, except at the ten dimensional boundaries of the orbifold interval. The boundaries therefore act as sources of the anomaly equation in eleven dimensions. The objects which live on the ten dimensional subspaces which we eventually refer to as instantons in the string theory carry unit charge. In addition to

the conventional placing of instantons, we can also distribute the charge amongst some M–theory fivebranes, which naturally carry (unit) charge also[13]. So when we compactify on $K3$, which supplies the opposite charge equal to 24 via geometry, we have an equation:

$$n_1 + n_2 + n_T = 24, \tag{3.1}$$

where n_1 and n_2 are the instanton contribution from the ten dimensional spacetimes at the end of the $S^1/\mathbf{Z}_2$ interval, while n_T is the number of M–theory fivebranes present. Of course, now we can see the dual nature of the term 'fivebranes' from our M–theory point of view. When they are living at the ten dimensional fixed points, they are the more conventional fivebranes which we recognized as the fully dressed string theory instantons (made of collections of D5–branes), and away from the the fixed points they are the eleven dimensional M–theory theory fivebrane. Where ever they happen to be in the higher dimensions, for the purposes of this consistent $K3$ compactification, the fivebranes must be transverse to the compact space. This means that their world volume is aligned with the non–compact directions $\{X^0, \ldots, X^5\}$.

It is no accident that we denoted the number of fivebranes in the interior of the interval in X^{10} by n_T. This is because the contribution of the fivebrane's worldvolume to the six dimensional spectrum is one tensor multiplet of $\mathcal{N}=2$ supersymmetry[9][13]: $(\mathbf{3},\mathbf{1})+5(\mathbf{1},\mathbf{1})$. In our $\mathcal{N}=1$ context, this is a contribution of a tensor multiplet $(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{1})$ and a hyper-multiplet $4(\mathbf{1},\mathbf{1})$, precisely what the closed string sector supplies from the $\mathbf{Z}_N$ orientifold fixed points ($N \neq 2$), as computed in ref.[11].

We are now in good shape to begin trying to understand how our other orientifold models might fit into the M–theory picture. Let us first try to understand precisely what models we wish to consider. In the case of the models $\mathbf{Z}_N^A$ which we are considering (for N even), there is a more complicated orientifold fixed point structure, as discussed in ref.[11]. However, there is one simplifying observation[11]. Regardless of the structure of the fixed points, in each case there is only one type of fixed point which is responsible for acting as a source of (*untwisted*) R-R 6–form charge (under discussion here) and that is the $\mathbf{Z}_2$ fixed point, the spacetime manifestation of the element ΩR in the orientifold group. (R denotes spacetime reflection in the X^6, X^7, X^8, X^9 directions, which forms a $\mathbf{Z}_2$ subgroup of all the models under consideration here). There are always 16 of these $\mathbf{Z}_2$ fixed points , which is why there are always 32 D5–branes. In other words, each such fixed point carries -2 units of D5–brane charge, as before[10].

---

[9] Here we denote the spacetime transformation properties of states by their dimension as a representation of the six dimensional little group $SU(2){\times}SU(2)$.

[10] The difference between the models arises when we realize that the $\mathbf{Z}_N$ orbifold acts by grouping N pairs of D5–branes into dynamical fivebrane units, making fewer of them available as N increases. They are paired because[2] of the presence of Ω.

We can therefore construct in each of these orientifold models the same special configuration yielding local cancellation of the untwisted R-R 6–form charge. Arguments analogous to those in ref.[6] ensure that the naive $U(1)^{16}$ gauge group is again broken completely, leaving only uncharged hypermultiplets and the tensor multiplets. The spectrum is then easy to determine for each model, and is listed below in summary:

Model	n_H	n_T
$\mathbf{Z}_2^A$	244	0
$\mathbf{Z}_4^A$	128	4
$\mathbf{Z}_6^A$	70	6

$$(3.2)$$

Turning back to the M–theory interpretation, if we interpret the tensors as coming from pushing fivebranes out into the interior of the eleven dimensional interval, then we are left with the task of distributing the remaining instanton number amongst the E_8 gauge groups when we compactify. We have certain constraints on how we can distribute if we wish to find our models. We must find configurations which give us a spectrum which can be Higgs–ed away to nothing, which is the gauge content of all of our models, as chosen above. Embedding instanton number 8 into E_8 for an $SU(2)$ gauge bundle leaves gauge group E_7 with 2 hypermultiplets in the **56**, by the index formula. This is not enough matter to break the group completely, leaving an unbroken $SO(8)$. Smaller amounts of instanton number produce larger unbroken gauge groups, and so we must consider instanton number greater than 8. Instanton numbers 12, 11 and 10 produce 4, $3\frac{1}{2}$ and 3 **56**'s respectively, and are known to have sequential Higgs–ing routes which lead to completely broken gauge groups. Instanton number 9 is interesting, however. By the index formula, it produces $2\frac{1}{2}$ **56**'s, which is (naively) enough to break the E_7 gauge group completely (since $140 > 133$). However, it seems that the Higgs–ing route to a completely broken gauge group has not yet been found. However, as pointed out in ref.[4], failure to find a Higgs–ing route does not rule out the existence of such a branch of moduli space. We shall assume that it exists, and our motivation will simply be that it fits all the available data of our models.

With this information in mind, we have the following candidate arrangements for our models. $\mathbf{Z}_4^A$ has $(n_1, n_2) = (10, 10)$ or $(11, 9)$, while $\mathbf{Z}_6^A$ has $(9, 9)$ as the only possibility. Given the data and techniques we have to work with, we have no way to decide between the two choices[11] for $\mathbf{Z}_4^A$. However, there is the possibility that the choices are related in the same way that the (12,12) and (10,14) models are related.

Another way to look at things is to examine how the spectra we found above match what

---

[11] The choice (10,10) seems more aesthetically pleasing, as it matches the (9,9) and (12,12) of the other models.

we would naively expect from the instanton moduli space. For large instanton number n, the index formula gives us an expression for the dimension of the moduli space $\mathcal{M}_n$ of E_8 instantons on $K3$:

$$\dim\mathcal{M}_n(E_8) = 120n - 992. \tag{3.3}$$

Putting in the numbers for each case (n_1, n_2), and dividing the result by 4 to get the number of hypermultiplets, we find the result 224 for $\mathbf{Z}_2^A$, 104 for $\mathbf{Z}_4^A$ and 44 for $\mathbf{Z}_6^A$. This is consistent with the above table of hypermultiplets if we add 20 hypermultiplets in each case for $K3$ gravitational moduli, and n_T hypermultiplets in each case corresponding to the positions[12] of the fivebranes in the $K3$.

We also wished to gain some insight into the importance of the number 8 in this setting. Now we see that it is simply the total number of M–theory fivebranes in the problem. In each of the orientifold models, the $K3$ manifold as an orbifold acts as a source of -24 units of 6–form charge in equation (3.1), (from geometry) and $+16$ units from instanton charge, localised in the 16 $\mathbf{Z}_2$ fixed points which are present. The other fixed points do not contribute to the counting as they have no 6–form charge. We expect that this distribution of instanton number is preserved in going to the M–theory compactification on smooth $K3$. So there, in order to cancel the remaining -8 units of charge, we have to introduce 8 fivebranes.

The order N of the spacetime symmetry of the orientifold models determines in M–theory how many fivebranes live on the ten–dimensional spacetime of an $S^1/\mathbf{Z}_2$ orbifold fixed point, playing the role of string theory instantons/mulitple D5–branes, leaving the rest on the X^{10} orbifold interval.

We have thus found a natural setting in which to place the orientifold models $\mathbf{Z}_2^A, \mathbf{Z}_4^A$ and $\mathbf{Z}_6^A$, which 'explains' their similarities and differences[13]. We expect that these models are all connected by a phase transition (from the six dimensional point of view) occurring when a fivebrane detaches from the ten dimensional world volume and goes into the bulk of the eleven dimensional spacetime[14,15]. This process lies naturally outside the description of perturbative heterotic string theory, which is why we had such difficulty interpreting the duals of the models inside a string theory framework (other than the type IIB orientifold framework).

We should note here that if we really want to cling to the idea of string theory, we can

---

[12] The movement of the M–theory fivebranes along the interval does not correspond to a true modulus of the theory. This is in line with the fact that the associated scalar is in a tensor multiplet and not a hypermultiplet.

[13] We should also mention that an M–theory realisation of a model closely related to the $\mathbf{Z}_4^B$ model[27] was worked out in ref.[28]. The eight extra tensors are produced by eight M–theory fivebranes, required by the presence of fixed points of an orbifold of smooth $K3$ by the Enriques involution.

naively take the small $S^1/\mathbf{Z}_2$ limit[14] of the M–theory construction to recover the $E_8{\times}E_8$ $K3$ compactified heterotic string and interpret the n_T extra tensors as 'new non–perturbative data', as we suggested at the outset[15]. The T_{6789} self duality of the orientifold models would then induce the action of a generalisation of the heterotic/heterotic duality for these string theories. The explicit construction of the fundamental strings which would get exchanged under this duality as solitons in our orientifold theories should be straightforward. One such string is simply the D1–brane, which we can arrange to live in the non–compact directions. The other is a D5–brane wrapped around the $K3$, also giving a strings in the non–compact directions, T_{6789}–dual to the D1–brane[6].

4. Some Four Dimensional Dualities

Let us compactify our M–theory models further on a torus T^2. Now we are in a four dimensional setting, with $\mathcal{N}{=}2$ supersymmetry. Recall that the six dimensional tensor multiplets *as well as* the vector multiplets give rise to four dimensional vector multiplets. Meanwhile hypermultiplets map to hypermultiplets, and the compactification on the extra torus gives us an extra four vector multiplets for gauge group $U(1)^4$.

So now we have three models with spectra which are given by the same number n_H of hypermultiplets as given in (3.2), with $n_V{=}n_T{+}4$.

This situation resembles something else that we have seen before. In ref.[29], a number of four dimensional $\mathcal{N}{=}2$ vacua were constructed by compactifying the heterotic string on $K3{\times}T^2$, with a special choice of gauge bundle. In addition to embedding instanton numbers (n_1, n_2) into each E_8, there was an embedding of n_T units into the non–Abelian gauge group obtained by placing the torus T^2 at a special point in its moduli space. Consistency was achieved by ensuring that the sum (3.1) was satisfied.

In the resulting four dimensional setting, there is no way of knowing whether the spectrum has originated from such a $K3{\times}T^2$ heterotic string compactification such as that carried out in ref.[29], or a $K3{\times}T^2{\times}S^1/\mathbf{Z}_2$ M–theory compactification with fivebranes, as described here. This is suggestive of a new and interesting duality relationship between the two, which deserves further exploration. It probably involves a map between the geom-

---

[14] Well, something more like a multiple scaling limit where we take the interval size to zero while holding the positions of the n_T fivebranes away from the interval's edges.

[15] There must be a T–duality relationship (generalising the one in ref.[6]) between the resulting $E_8{\times}E_8$ string theory and the $SO(32)$ one, realising the scenario (**2**) described in the first part of section 2. Although probably complicated, due to the presence of the extra tensors, it should exist, given the fact that the A–type orientifold models are locally $SO(32)$ type I theory. We thank Joe Polchinski for pointing out this possibility to us.

etry of the special torus (with its gauge bundle of instanton number n_T) and the eleventh dimension's orbifold interval (and its n_T fivebranes)

Instead of following that avenue of investigation, there is yet another duality which is relevant in this setting, which was the subject of ref.[29]. The heterotic vacua we have just constructed in four dimensions are described at strong coupling by a compactification of type IIA string theory on Calabi–Yau 3–folds with the Hodge numbers $h_{2,1}=n_H-1$ and $h_{1,1}=n_T+3$, defining for us three 3–folds Y_1, Y_2 and Y_3:

C–Y 3–fold	$h_{2,1}$	$h_{1,1}$
Y_1	243	3
Y_2	127	7
Y_3	69	9

$$(4.1)$$

Of course, Y_1 is already extremely well known, in this and other related contexts and is the 3–fold denoted $X_{24}(1,1,2,8,12)$ earlier. The other two are not known to us at the time of writing. However, we expect that they exist, and furthermore that they exist as $K3$ fibrations over $\mathbb{P}_1$, where the size of the base determines the strength of the heterotic string coupling, a basic result of heterotic/type IIA duality[30]. We will ask some more of them in the next section.

5. F–Theory

In the previous section, by compactifying on T^2 and invoking four dimensional heterotic/type IIA duality, we arrived at type IIA string theory vacua. This is amusing, since we started out by orientifolding type IIB string theory, and we might wonder whether we can complete the circle of dualities, perhaps learning more along the way.

Well, the standard route from here to get to type IIB string theory would be perhaps to use mirror symmetry, and study a compactification on the mirrors of the manifolds in (4.1). This is not an attractive route for the interests of this paper for at least two reasons. The first is that we know so little about the manifolds Y_2 and Y_3, and so the investigation would be rather short. The second is that it is will not obviously lead us back to a six dimensional type IIB setting to complete the circuit.

The route we wish to take was already deduced in refs.[7,9]. Starting with type IIA string theory compactified on a Calabi–Yau manifold X, one can imagine a limit (loosely, X has to be large) in which the strong coupling limit might be captured by M–theory on $S^1 \times X$. From here, we can use a number of conjectured dualities to go into almost any direction

15

we want. For example, we can exploit the fact that X is a $K3$ fibration and try deduce a duality fibre by fibre by wrapping the M–theory fivebrane on $K3$ eventually deducing again the relation to heterotic strings. Another route is to require that X has an ellitpic fibration, compactify on the resulting torus to nine dimensions where we can T–dualise to type IIB theory, and from there go on to F–theory on $X \times S^1 \times S^1$. When we let this new two torus grow large, we have F–theory compactified on the 3–fold X.

F–theory compactified on a Calabi–Yau 3–fold gives six dimensional $\mathcal{N}=1$ vacua that are related to heterotic string vacua as first set out in refs.[9], by relating it to the Y_1 example of heterotic/type IIA and heterotic/heterotic duality.

Following that route we see that there is a natural interpretation in F–theory of many other four dimensional $\mathcal{N}=2$ vacua existing as dual heterotic/type IIA pairs. The associated Calabi–Yau 3–fold would have to be elliptic, in order to have a six dimensional interpretation when used as an F–theory compactification. The six dimensional vacua will contain extra tensors, the number of which is given by $h_{1,1}(B)-1$ where B is the base of the elliptic fibration of the 3–fold[9].

So the extra requirement we ask of our manifolds in (4.1) is that they are all elliptic, yielding for us new F–theory backgrounds with spectra given in (3.2).

6. A Return to Orientifolds

Until we learn more about F–theory, we are free to regard it as a new way of learning about backgrounds for the type IIB string. In this sense, it is very much akin to the orientifold technology, and it would be nice to make something of this.

The torus of the twelve dimensions of F–theory is a geometrisation of the coupling 'constant' of type IIB theory. On compactifying on an elliptic manifold to some dimension lower than ten, we are constructing a compactification of type IIB strings on the base B of the fibration. For example, in the case of compactification to six dimension by means of an elliptic Calabi–Yau 3–fold, the base B is not itself Calabi–Yau, and so we have naively obtained a sick IIB background. However, there are sevenbranes present in the problem, their positions given by the locations in the base B where the torus fibre degenerates. The presence of the sevenbranes completes the consistency requirement for yielding a IIB background.

That the sevenbranes are present is very natural. The torus fibre parameterizes the IIB coupling via the modulus $\tau(z_i)=A_0+e^{-\frac{\Phi}{2}}$ where Φ is the dilaton and A_0 is the R-R 0–form (scalar) potential. The z_i are coordinates on the base B over which the torus T^2 is fibred. The $SL(2, \mathbf{Z})$ self duality symmetry of the type IIB string acts on the torus. Recall that the sevenbrane is the natural object in the theory carrying electric charge of (the field strength of) A_8 and hence magnetic A_0 charge. The degeneration of the torus at positions on the base is simply a signal the presence of a magnetic source of A_0, the sevenbrane[7].

Consistency requires a certain number of sevenbranes to be present in order to correctly cancel the A_0 charge. We have heard this story three times before in this paper. Once in the context of orientifolds and D–branes, once in the context of heterotic string theory and consistent $K3$ compactifications with instantons and once more in M–theory in the context of combining $K3$ compactifications with fivebranes. We discussed the relations between them. We should therefore expect that this charge cancellation is once again the orientifold charge cancellation in disguise and we shall see that it is.

In ref.[10], a precise relation between the construction of F–theory on a smooth elliptic $K3$ manifold and a type IIB orientifold was made. This construction enabled a more precise demonstration that the resulting background was dual to heterotic string compactified on T^2. The $K3$ is fibred as T^2 over a base $\mathbb{P}_1$. Generically, the fibre degenerates at 24 positions, implying that there are that many sevenbranes in the problem, their worldvolumes aligned with the uncompactified eight dimensions with a point–like intersection on the $\mathbb{P}_1$. At this stage, it is not quite true to say that this is a type IIB string theory background obtained by compactifying on $\mathbb{P}_1$ and including 24 branes, as stressed in ref.[7]. Clearly, the string theory description could not be perturbative, as the coupling is varying all over the $\mathbb{P}_1$.

It was demonstrated in ref.[10] however, that a limit could be approached where the torus parameter $\tau(z)$ does not vary smoothly over the base but is constant with the possibility of phases at a finite number of positions. In that limit, the type IIB string theory description can be made to work. It is a type of $\mathbf{Z}_2$ orientifold of the theory on a torus T^2. The orientifold requires 32 D7–branes to be in the problem, for charge cancellation. The four fixed points of the orientifold have charge -8 in D7–brane units and the charge cancellation can be carried out locally by placing 8 D–branes at each of the points.

As mentioned before, F–theory on the 3–fold $Y_1 = X_{24}(1,1,2,8,12)$ is another realisation of the (12,12) $K3$ heterotic compactification, which in turn has a realisation as the $\mathbf{Z}_2^A$ orientifold model.

It is natural to wonder if we can directly find a relation between the F–theory compactification and the orientifold, along the lines of ref.[10]. It is easy to see the relation if we begin by T–dualising along two of the directions of the orientifold four torus. Let us choose directions X^6 and X^7. Recalling that T–duality exchanges Dirichlet and Neumann boundary conditions, we see that the D5–branes get converted to D7–branes with world volumes located along direction X^μ for $\mu \in \{0,1,2,3,4,5,6,7\}$, while the D9–branes get converted to D7–branes located along the directions given by $\mu \in \{0,1,2,3,4,5,8,9\}$ directions.

The presence of the D7–branes is already heartening, in view of their natural occurrence in F–theory. Examining the details of the duality more carefully, we see that we are carrying out an orientifold of the torus $T^2 \times T^2$ with group

$$G = \{1, R_{6789}, \Omega R_{67}(-1)^{F_L}, \Omega R_{89}(-1)^{F_L}\}. \tag{6.1}$$

Geometrically the $\mathbf{Z}_2$ actions on the tori are given by the reflections R_{67} and R_{89} respec-

tively. Of course, $R_{6789}=R_{67}\cdot R_{89}$. The appearance of $(-1)^{F_L}$, where F_L is the left fermion number can be traced to the action of T–duality as an action of world sheet parity which is restricted to only the right or left movers[31,12]. Our conventions are such that the action is on the left.

We should expect that the orientifold torus $T^2 \times T^2$ is related to the base $\mathbb{P}_1 \times \mathbb{P}_1$ of the elliptic manifold of the smooth description in the same way that the orientifold torus T^2 was related to the base $\mathbb{P}_1$ in the eight dimensional example[10]. Let us see how this works.

In the eight dimensional example, a Weierstrass representation was used for the elliptic fibration of $K3$ as a torus T^2 over the sphere $\mathbb{P}_1$. There is a standard extension to an elliptic fibration over $\mathbb{P}_1 \times \mathbb{P}_1$:

$$y^2 = x^3 - f(z_1, z_2)x - g(z_1, z_2), \tag{6.2}$$

giving the 3–fold $X_{24}(1, 1, 2, 8, 12)$ when f and g are polynomials of degree 8 and 12 in the z_i, which are coordinates of the $\mathbb{P}_1$'s. Counting parameters, one can verify existence of the 243 complex deformations, after taking into account the rescaling freedom (on the affine coordinates x and y) and the $SL(2,\mathbb{C}) \times SL(2,\mathbb{C})$ symmetry. The symmetry exchanging the two $\mathbb{P}_1$'s is also manifest, translating into heterotic/heterotic duality as mentioned previously[9].

At any point on the base B, the $K3$ fibration is also clearly visible, the fibre itself being elliptic. The worldvolume of the sevenbranes on the base B is given by the vanishing of the discriminant

$$\Delta = 4f^3 - 27g^2, \tag{6.3}$$

an equation which has 24 solutions, generically.

In the work of ref.[10], a special point in the moduli space of the $K3$ was chosen such that the modular parameter of the torus fibre $\tau(z)$ (determined from the above representation implicitly in terms of the elliptic j–function) is independent of z. Here, z is the coordinate of the $\mathbb{P}_1$ base of the $K3$ fibration. We can take it to be either z_1 or z_2 here, and the analysis will go through with the other $\mathbb{P}_1$ remaining a spectator. In this limit, the discriminant Δ takes a form which indicates that the 24 sevenbranes have coalesced into four groups of six coincident branes, located around four fixed points z_1, z_2, z_3, z_4. The parameter τ is constant over the base, with a non–trivial $SL(2, \mathbf{Z})$ monodromy around each of the points.

Furthermore, the metric of the base was computed in this limit, and it turns out to be globally a flat geometry, together with a deficit angle of π at each of the four special points where the sevenbranes are located.

The orientifold interpretation of this scenario[10] is that the monodromy around the points is $(-1)^{F_L}\Omega$ while the geometry of each point is that of a $\mathbf{Z}_2$ fixed point of a spacetime reflection action on a two torus. In other words, the orientifold group element is $\Omega R_{89}(-1)^{F_L}$, which we see appearing in the T_{67} dual of the $\mathbf{Z}_2^A$ model defined by the orientifold group in

18

eqn. (6.1). In completing the rest of such an orientifold model, tadpole cancellation would require the addition of open string sectors in the form of 32 D7–branes. A computation would reveal that there are -8 units of D7–brane charge on each of the four fixed points and the condition for local cancellation of the charge (associated to the field strength of the R-R 8–form A_8, dual to A_0, part of the coupling τ) is to group them into four groups of 8 coincident branes[16]. This final configuration matches that of the special F–theory configuration.

So far, we have simply forgotten about the other component of the base, the $\mathbb{P}_1$. It is simply brought into the discussion by carrying out the same procedure again, this time forgetting about the first $\mathbb{P}_1$. As the base is simply a product of the $\mathbb{P}_1$'s there is nothing lost in this piecewise approach. This time we end up with the same orientifold story, with coordinates $\mu \in \{6, 7\}$ instead of $\{8, 9\}$. Having thus deduced the presence of the operation $\Omega R_{67}(-1)^{F_L}$, then we are forced by closure to have $R = R_{6789}$ in the orientifold group too.

Thus we see that we have recovered the orientifold group (6.1) we deduced from T_{67}–dualising the $\mathbf{Z}_2^A$ model, forging another connection between F–theory on $X_{24}(1, 1, 2, 8, 12)$ and heterotic/heterotic duality.

The next task (beyond the scope of this paper) would be to carry out the same procedure for the models $\mathbf{Z}_4^A$ and $\mathbf{Z}_6^A$ starting with the manifolds Y_2 and Y_3. The procedure of T_{67} dualising is obvious. However, the orientifold action on the $\mu \in \{6, 7, 8, 9\}$ torus T^4 will not be factorisable, as one might expect from the fact that the base manifold of the associated (conjectured) elliptic fibrations of the 3–folds Y_2 and Y_3 in (4.1) is unlikely to be a trivial product. This is especially true since it must have $h_{1,1}$ large enough to yield non–zero n_T in the final six dimensional spectrum.

Despite that complication, we would expect to see an involution of the base B of the manifolds Y_2 and Y_3, which provide a geometrical realisation of the exchange symmetry T_{6789}. This would be the map which generalizes the heterotic/heterotic duality map of the simpler model. It is worth exploring what the consequences of such a map would be for the M–theory compactifications and related theories.

7. Conclusions

We have come full circle in our exploration of a chain of dualities. We started with the orientifold models of refs.[2,11], and related special cases of them to M–theory compactifications to six dimensions on $K3 \times S^1 / \mathbf{Z}^2$ with extra fivebranes. This extends the correspondence

---

[16] The attentive reader may have by now noticed a discrepancy of a factor of two between the counting of ref.[10] and the counting here. All is well, for ref.[10] counts a D–brane and its mirror as one object, while we count them as two.

found in ref.[6] to the $K3$ compactified $(12, 12)$ $E_8 \times E_8$ heterotic string for the case with no extra tensors.

Going to four dimensions via the torus T^2, we noted that the spectra found there can be viewed as having a dual origin, either from M–theory with extra fivebranes or from heterotic strings with instantons embedded in the gauge group arising from placing the torus at a special point. That relationship deserves further exploration.

We used four dimensional heterotic/type IIA duality to deduce what properties two new (to the authors) Calabi–Yau 3–folds would need to have to be associated with the original orientifold models.

Being in type IIA string theory it was natural to try to seek a type IIB relationship, which led us to F–theory. From there, for the model for which we have all of the data on the Calabi–Yau 3–fold, we were able to simply extend the ideas of ref.[10] down a further two dimensions to recover a T–dual of the orientifold model we first started with. We have thus found a direct relationship to F–theory's smooth description. We expect that this works for the other two orientifold examples we considered in this paper.

This completes our instructive tour of the duality circuit, illustrating many links between ideas. Many of the links were organised by (or have a simple interpretation in) the framework of M– and F–theory, the parent theories from which all string theories seem to originate.

Note Added:

While preparing this manuscript for publication, ref.[32] appeared, in which related work is presented.

References

[1] E. Witten, *'Small Instantons in String Theory'*, preprint IASSNS-HEP-95-87, hep-th/9511030.

[2] E. G. Gimon and J. Polchinski, *'Consistency Conditions for Orientifolds and D–Manifolds'*, hep-th/9601038.

[3] E. Witten, *'String Theory Dynamics in Various Dimensions'*, Nucl. Phys. **B443** (1995) 85, hep-th/9503124.

[4] M. J. Duff, R. Minasian and E. Witten, *'Evidence for Heterotic/Heterotic Duality'*, hep-th/9601036.

[5] M. J. Duff and J. X. Lu, *'Loop Expansions and String/Fivebrane Duality'*, Nucl. Phys. **B357** (1991) 534;
M. J. Duff and R. R. Khuri, *'Four–Dimensional String/String Duality'*, Nucl. Phys. **B411** (1994) 473, hep-th/9305142;
M. J. Duff and R. Minasian, *'Putting String/String Duality to the Test'*, Nucl. Phys. **B436** (1994) 507, hep-th/9406198

[6] M. Berkooz, R. G. Leigh, J. Polchinski, J. H. Schwarz, N. Seiberg and E. Witten, *'Anomalies, Dualities, and Topology of D=6 N=1 Superstring Vacua'*, hep-th/9605184.

[7] C. Vafa, *'Evidence for F–Theory'*, hep-th/9602022.

[8] G. Aldazabal, A. Font, L. E. Ibáñez and F. Quevedo, *'Heterotic/Heterotic Duality in D=6, 4'*, hep-th/9602097.

[9] D. R. Morrison and C. Vafa, *'Compactifications of F–Theory on Calabi–Yau Manifolds I & II'*, hep-th/9602114 & hep-th/9603161.

[10] A. Sen, *'F–Theory and Orientifolds '*, hep-th/9605150.

[11] E. G. Gimon and C. V. Johnson, *'K3 Orientifolds'*, hep-th/9604129.

[12] A. Dabholkar and J. Park, *'Strings on Orientifolds'*, hep-th/9604178.

[13] E. Witten, *'Fivebranes and M–Theory on an Orbifold'*, hep-th/9512219.

[14] O. J. Ganor, A. Hanany, *'Small E_8 Instantons and Tensionless Non–critical Strings'*, hep-th/9602120.

[15] E. Witten, *'Phase Transitions in M–Theory and F–Theory'*, hep-th/9603150;
N. Seiberg and E. Witten, *'Comments on String Dynamics in Six Dimensions'*, hep-th/9603003.

[16] M. J. Duff, H. Lu and C. N. Pope, *'Heterotic Phase Transitions and Singularities of the Gauge Dyonic String'*, hep-th/9603037.

[17] E. Witten, *'Some Comments on String Dynamics'*, hep-th/9507121.

[18] A. Strominger *'Heterotic Solitons'*, Nucl. Phys. **B343** (1990) 167;
C. G. Callan, J. A. Harvey and A. Strominger, *'Solitons in String Theory'*,, Trieste Notes, World Scientific (1991), hep-th/9112030.

[19] J. Polchinski, *'Tensors from K3 Orientifolds'*, hep-th/9606165.

[20] P. Hořava and E. Witten, *'Heterotic and Type I String Dynamics from Eleven Dimensions'*, Nucl. Phys. **B460** (1996) 506, hep-th/9510209

[21] M. B. Green and J. H. Schwarz, *'Anomaly Cancellation In Supersymmetric D=10 Gauge Theory and Superstring Theory'*, Phys. Lett. **B149** (1984) 117.

[22] A. Sagnotti, *'A Note on the Green–Schwarz Mechanism in Open String Theories'*, Phys. Lett. **B294** (1992) 196, hep-th/9210127.

[23] T. Eguchi and A. J. Hanson, *'Asymptotically Flat Self–Dual Solutions to Euclidean Gravity'*, Phys. Lett. **B74** (1978), 249.

[24] M. B. Green, J. H. Schwarz and P. C. West, *'Anomaly Free Chiral Theories in Six Dimensions'*, Nucl. Phys. **B254** (1985) 327.

[25] J. Polchinski and E. Witten *'Evidence for Heterotic/Type I Duality'*, preprint Nucl. Phys. **B460** (1996) 525, hep-th/9510169.

[26] K. Dasgupta and S. Mukhi, *'Orbifolds of M–theory'*, hep-th/9512196.

[27] A. Dabholkar and J. Park, *'An Orientifold of Type–IIB Theory on K3'*, hep-th/9602030.

[28] A. Sen, *'M-Theory on $(K3 \times S^1)/\mathbf{Z}_2$'*, hep-th/9602010.

[29] S. Kachru and C. Vafa, *'Exact Results for N=2 Compactifications of Heterotic Strings'*, Nucl. Phys. **B450** (1995) 69, hep-th/9505105.

[30] A. Klemm, W. Lerche and P. Mayr, *'K3–Fibrations and Heterotic–Type II String Duality'*, hep-th/9506112.

[31] J. Polchinski, S. Chaudhuri and C. V. Johnson, *'Notes on D–Branes'*, hep-th/9602052.

[32] P. Candelas, E. Perevalov and G. Rajesh, *'F–Theory Duals of Nonperturbative Heterotic $E_8 \times E_8$ Vacua in Six Dimensions'*, hep-th/9606133.